MATH 021B DISCUSSION WORKSHEET 3 (MIDTERM REVIEW)

Problem 1. Fill in the following table with useful derivatives and antiderivatives:

$f(x)$		$\sin(x)$			$\arctan(x)$	$\cot(x)$		
$f'(x)$	x^n		$\sec^2(x)$	a^x			$\frac{1}{x\sqrt{x^2-1}}$	$\frac{1}{x}$

Problem 2. Compute the indefinite integrals, without forgetting the $+C$:

(a) $\int \sin(x)(\cos(x)-2)^5 \mathrm{d}x$

(b) $\int \left(\frac{5}{x\sqrt{(x^2-9)}} - \frac{\sec^2(x)}{\tan(x)-10} \right) \mathrm{d}x$

(c) $\int \frac{2\arctan(\ln(x))}{x((\ln(x))^2+1)} \mathrm{d}x$

(d) $\int \sin^2(\pi) \ln\left(\frac{1}{1+x^4}\right) \mathrm{d}x$

[Hint: What is $\sin(\pi)$?]

Problem 3. Compute the area between the curves $y = \sin(x)$ and $y = \frac{3}{5\pi}x$. [Hint: One of the points where the curves intersect has y-value $\frac{1}{2}$.]

Problem 4. Consider the region bounded by the lines $x = 0$, $x = 4$, $y = 10 - x$, and the x-axis. Find the value of c so that the line $y = c$ divides this region into two regions of equal area. [Bonus Challenge: What if you replace the line $y = 10 - x$ with the line $y = 10 - 2x$?]

Problem 5. Find the derivatives of the following functions:

(a) $f(x) = \int_{x^2}^{10} \sin(\ln(t)) \mathrm{d}t$

(b) $g(x) = \int_{\sin(x)}^{e^{-x}} 10 \, \mathrm{d}t$

Problem 6. (a) Write down (and do NOT evaluate) an expression using sigma notation for the Riemann sum of $f(x) = x^3 - x$ on $[0, 4]$ using midpoints and with $n = 4$.

(b) Sketch the graph of $f(x)$, along with the rectangles described by your Riemann sum from part (a).

(c) Write down an expression in sigma notation for the Rieman sum of $f(x) = x^3 - x$ on $[0, 4]$ for n many rectangles using right endpoints.

(d) Use the summation formulae from the formula sheet to rewrite your answer from (c) without sigma notation.

(e) Compute the limit as $n \to \infty$ of your answer from (d).

(f) Check your answer from (e) by computing the definite integral $\int_0^4 (x^3 - x)\mathrm{d}x$.

Warning: The following problems are extra difficult. The ultimate form of practice!

Problem 7. (Challenge Problem) Evaluate the limit:

$$\lim_{n\to\infty} \frac{\sin\left(\frac{\pi}{n}\right) + \sin\left(\frac{2\pi}{n}\right) + \cdots + \sin\left(\frac{n\pi}{n}\right)}{n}.$$

[Hint: Try to express the fraction as a Riemann sum.]

Problem 8. (Challenge Problem) Evaluate the integral:

$$\int_0^1 y \cdot \left(\int_0^{y^2} \left(e^x + \sec^2(x)\right) \mathrm{d}x \right) \mathrm{d}y.$$

[Hint: Start with the integral in parentheses and take it one step at a time, you got this!]

Good luck on the exam! Try your best, and don't forget the $+C$.