MATH 21B MIDTERM 3 REVIEW

Problem 1. For this problem, consider the function

$$f(x) = x\sin(x)$$

on the interval $[0, 4]$. Use the following table of approximate values:

x	0	1	2	3	4
$x\sin(x)$	0	0.8	1.8	0.4	-3.0

(a) Approximate the integral

$$\int_0^4 x\sin(x)dx$$

using the trapezoid method with $n = 4$.

(b) Give an upper bound for the error in the approximation from part (a).

(c) Approximate the same integral using Simpson's approximation with $n = 4$.

(d) Give an upper bound for the error in the approximation from part (c).

(e) Solve the integral to obtain an exact solution. Which approximation was closer? Confirm that the exact value is within the margins of error you calculated in (b) and (d).

Problem 2. More practice integrals. Some are improper integrals: for the improper integrals, determine if they converge or diverge. If they converge, compute the integral.

(1) $\displaystyle\int \frac{2e^x}{e^{2x}-1}dx$

(2) $\displaystyle\int (x^2+x+1)\ln(x)dx$

(3) $\displaystyle\int_3^\infty \frac{2}{(3x-6)^3}dx$

(4) $\displaystyle\int_2^\infty \frac{2}{(3x-6)^3}dx$

(5) $\displaystyle\int \frac{s}{(s-1)(s^2-1)}ds$

(6) $\displaystyle\int \frac{\sqrt{y+9}}{\sqrt{y-16}}dy$

(7) $\displaystyle\int \sin^4(x)\cos^2(x)dx$

(8) $\displaystyle\int \sin^4(x)\cos^3(x)dx$

(9) $\displaystyle\int e^{-x}\cos(2x)dx$

(10) $\displaystyle\int t^3\sin(t^2)dt$

(11) $\displaystyle\int \sec^5(\theta)d\theta$

(12) $\displaystyle\int \frac{z^5}{\sqrt{1+z^4}}dz$